Combined Design and Control Optimization
What is Optimal Design?

- Artifact
- Embodiment
- Mechanism
- Sizing
- Topology
- Static

Variables: \( d \)

Parameters: \( c \)

Outputs: \( f, g, h \)

Metric: \( f(d;c) \)

Variables: \( d \in D \subseteq \mathbb{R}^q \)

Parameters: \( c \)

subject to:

\( h(d;c) = 0 \)

(GDP)

\( g(d;c) \leq 0 \)
What is Optimal Control?

- Dynamic Response Control Strategy Gains

Control

(states $x(t)$ determined)

Parameters: $w$

Variables: $z(t)$ or $p$

Outputs: $J$, $k$, $l$

Metric:

$$J = \int_{0}^{t_f} j(x(t), z(t), t; w) dt$$

Variables:

$t \in [0, t_f]$, $x(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^m$

Parameters: $w$

subject to:

(GCP)

$$\dot{x} = r(x(t), z(t), t; w)$$

$$y = s(x(t), z(t), t; w)$$

$$k(x(t), z(t), t; w) = 0$$

$$l(x(t), z(t), t; w) \leq 0$$

$$z(t) = z(x(t), y(t), p, t; w)$$

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# Why Combine Design and Control?

- Systems
- Electromechanical
- Mechatronics

## Systems Viewpoint
- Interested in Optimal System
- Is an optimal design and an optimal control equivalent to an optimal system?

## Inherent Coupling via Parametric Dependencies
- Control Parameters depend on Design
- Design Parameters depend on Control
CONCURRENT PROBLEM

Design

Control

(states $x(t)$ determined)
Solution Methods

- Sequential
  - Single Pass (Traditional)
  - Iterative

- Combined
  - Fix input parameters, optimize sequential system
  - All At Once: crossover variables as parameters
  - Treat OGP as subproblem to overall optimization

Compare
- final solutions
- independence of starting points
- convergence
- ability to use known theories, ease of use
Sequential Methods

Single Pass Methods

\[ b_{in} \rightarrow \text{minimize } f \text{ \ variables: } d \text{ \ parameters: } b_{in} \text{ \ subj. to: } GDP \rightarrow v \rightarrow \text{minimize } J \text{ \ variables: } p \text{ \ parameters: } v \text{ \ subj. to: } GCP \rightarrow b_{out} \rightarrow \text{Verify}^* \]

\[ b_{out} - b_{in} \leq 0 \]

Iterative Methods

\[ b_{in} \rightarrow \text{minimize } f \text{ \ variables: } d \text{ \ parameters: } b_{in} \text{ \ subj. to: } GDP \rightarrow v \rightarrow \text{minimize } J \text{ \ variables: } p \text{ \ parameters: } v \text{ \ subj. to: } GCP \rightarrow b_{out} \rightarrow \]

\[ |b_{out} - b_{in}| \leq \text{tolerance or exceed max iterations} \]
Combined Methods

Fixed Input Parameter Combination

minimize $w_1 \cdot f + w_2 \cdot J$

Variables: $d, p$

Parameters: $b_{in}$

Subject to:

$\begin{align*}
  b_{in} & \rightarrow \text{solve} \quad \text{GDP} \\
  v & \rightarrow \text{solve} \quad \text{GCP} \\
  b_{out} & \leftarrow
\end{align*}$

Additional Constraints: $b_{out} - b_{in} \leq 0^*$

All At Once / Variable Input Parameters

minimize $w_1 \cdot f + w_2 \cdot J$

Variables: $d, p, b_{in}$

Subject to:

$\begin{align*}
  b_{in} & \rightarrow \text{solve} \quad \text{GDP} \\
  v & \rightarrow \text{solve} \quad \text{GCP} \\
  b_{out} & \leftarrow
\end{align*}$

Additional Constraints: $b_{out} - b_{in} \leq 0^*$
OGP Subproblem of System

minimize $w_1 f + w_2 J$

Variables: $d, p, v_{in}$

Subject to:

Additional Constraints: $v_{out} - v_{in} \leq 0$*

- No dependence on input crossover parameters
- Can use std. design and control optimization methods
DC Motor Problem

Have an application that requires a motor. Want to choose the ‘best’ motor, with a controller, for the job.

Design:
- Determine the design: dimensions of motor, size of armature, length of wires
- Objectives: minimize the weight of the motor
- Linking Parameters: motor speed, voltage, torque and power requirements

Control:
- Determine the controller: choose a control strategy (PID) and determine its gains.
- Objectives: minimize error of desired variable, minimize voltage requirement
- Linking Parameters: inductance, resistance, motor constant, rotor inertia
Solution Locations

DC Motor Pareto Surface

Design Objective – Motor Weight

Control Objective – Performance Index

High wt

Low wt

SP–DF

SP–CF

I–DF

I–CF

C–FIX

C–SUB

C–VAR

SP: Single Pass

I: Iterative

DF: Design First

CF: Control First

C-FIX: Fixed Input Parameters

C-VAR: All At Once

C-SUB: Optimal Control As System Subproblem
Dynamic Responses

SP: Single Pass
I: Iterative
DF: Design First
CF: Control First
C-FIX: Fixed Input Parameters
C-VAR: All At Once
C-SUB: Optimal Control As System Subproblem
# Constraint Activity

| Solution Method       | g1 | g2 | g3 | g4 | g5 | g6 | g7 | g8 | g9 | g10 | g11 | g12 | g13 | g14 | g15 | g16 | g17 | g18 | g19 | b1 | b2 | b3 | b4 |
|-----------------------|----|----|----|----|----|----|----|----|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|----|----|----|----|
| Single Pass           |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| Design - First        |    | * |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| Control - First       |    |   |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| Iterative             |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| Design - First        |    | * |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| Control - First       |    |   |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| Combined              |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| Fixed Crossover       |    | * |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| All At Once           |    |   |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |
| OGP Subprob           |    |    |    |    |    |    |    |    |    |     |     |     |     |     |     |     |     |     |    |    |    |    |

- $g_2$: magnetic saturation
- $g_3, g_4$: UB, LB of rotor's peripheral speed
- $g_5, g_6$: UB, LB of length to pole pitch ratio
- $g_{11}$: tooth width to slot depth ratio
- $g_{13}$: UB of specific electrical loading
- $g_{15}$: design power
- $g_{16}$: design torque
- $g_{18}, g_{19}$: LB, UB of permissible heat loss per unit of surface area
- $l_3$: positivity of derivative gain
- $b_4$: i/o of torque crossover parameter
Conclusions & Future Work

- Optimal design and optimal control need to be integrated to consider system optima.

- An All-At-Once approach may yield the best results, but we will not always be able to solve such a problem.

- Need to define cases when single pass or iterative techniques are insufficient

- Need to determine criteria for evaluating possible improvement from single pass method to all-at-once method, based on easily accessible results.