Simulating Track/Sprocket and Track/Wheel/Terrain Contact in Tracked Vehicles

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Objective

Efficient simulation of vehicle response including track vibration, track/sprocket contact, and track/wheel/terrain interaction.

- Overview of recent efforts
- New effort: Track/Sprocket Contact
- New effort: Track/Wheel/Terrain Interaction
Track Models

QUASI-STATIC

MULTI-BODY CHAIN

HYBRID DISCRETE-CONTINUOUS ELEMENT
Continuous Track Model

IN-PLANE RESPONSE

TRANSVERSE

LONGITUDINAL
**Continuous Track Model**

**Longitudinal Response (quasi-static)**

\[ u_1(s,t) = f(t)s + k \int_0^s u_2(\eta,t) d\eta \]

**Transverse Response**

\[ m(u_{2,tt} + 2Vu_{2,st} + V^2u_{2,ss}) + kEAf(t) = Tu_{2,ss} \]

\[ f(t) = \frac{-k}{L} \int_0^L u_2(\eta,t) d\eta \]

- \( V \) - track speed
- \( d \) - static sag
- \( k \) - static curvature
- \( T \) - static tension
- \( f(t) \) - dynamic strain

Longitudinal Response

- Quasi-static response

Transverse Response
Experimental Validation
M1 Tank Traversing Profile 4
Track Transverse Vibration
Including Track/Sprocket Contact

Objective: To model of dynamic shoe-tumbler seating process

- **Inputs**: shoe pitch elongation, tumbler lug wear, track tension, interface friction, applied torque, ground conditions.

- **Outputs**: pin loading, shoe/tumbler contact forces, animation of seating process
P&H 4100A Crawler

- Operating weight: 1,200 tons
- Max. speed: 0.56 mph.
- Max. applied torque: 1,580,000 ft-lbs
- Max. design tension: 1,480,000 lbs
Shoe/Tumbler Seating Process
Hybrid Track Model

- Multibody Model of Shoes
- Continuous Model of Track Segments
Shoe/Tumbler Interface

a) Slight Wear  b) Heavy Wear
Prescribed Tumbler Velocity
Machine Travel Speed
Shoe/Tumbler Normal Contact Force
For Shoe 3-7
Vertical Load For Pins 01 to 03 Under Rear Idler

![Graph showing vertical load for pins 01 to 03 under rear idler.](image-url)
Case: Slight Wear, Very Tight
Case: Heavy Wear, Extremely Loose
Including Track/Wheel/Terrain Contact

Objective: To develop track/wheel/terrain interface model to efficiently predict the loading on the vehicle due to the ground pressure and shear.
Track/Wheel/Terrain Interaction Model
(Force Element)

Inputs

$$(X_1, Z_1, \phi_1)$$
$$(X_2, Z_2, \phi_2)$$
$$z_t(x)$$

Outputs

$$(F_X^1, F_Z^1, \tau_1)$$
$$(F_X^2, F_Z^2, \tau_2)$$
Track/Wheel/Terrain Interaction

**Equilibrium**

\[-T \cos \theta + (T + dT) \cos(\theta + d\theta) - f_x ds = 0\]

\[-T \sin \theta + (T + dT) \sin(\theta + d\theta) - f_z ds = 0\]

**Kinematic Relationships**

\[\cos \theta = \frac{dx}{d\rho}, \quad \cos(\theta + d\theta) = \frac{dx}{d\rho} + d\left(\frac{dx}{d\rho}\right) ds\]

\[\sin \theta = \frac{dz}{d\rho}, \quad \sin(\theta + d\theta) = \frac{dz}{d\rho} + d\left(\frac{dz}{d\rho}\right) ds\]

**Constitutive Law**

\[T = EA(d\rho - ds) / ds\]
Contact Models

**Track/Wheel**

\[
f = \begin{cases} 
-k_w (r - r_1) & \text{for } r < r_1 \\
0 & \text{for } r \geq r
\end{cases}
\]

**Track/Terrain**

\[f_x = \tau + mg \sin \alpha \text{ and } f_z = p - mg \cos \alpha\]

\[
\tau(x, z) = \tau_m \left[1 - \exp\left(-\frac{|x-x_t|}{K}\right)\right] \quad \text{(Janosi & Hanamoto)}
\]

\[
\tau_m = C + p(z) \tan \phi
\]

\[
p(z) = \left(\frac{k_c}{W} + k_\phi\right)(z - z_t)^n \quad \text{(Bekker)}
\]
Solution Procedure

- Re-write BVP in incremental form
- Discretize by FE
- (Modified) Newton-Raphson Iteration of nonlinear BVP
Example: Case of Zero Sinkage

Trapezoid Bump

Round Bump
Example: Case of One Inch Sinkage
Example: Effect of Ground Stiffness

- $K=3.6 \times 10^4$
- $K=1.8 \times 10^4$
- $K=0.9 \times 10^4$
Effect of Track Extensibility

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<th>EA (lbs)</th>
<th>Tension (lbs)</th>
<th>Fx (lbs)</th>
<th>Fz (lbs)</th>
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## Continuous to Multibody Formulation

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<tr>
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<th>FX (lbs)</th>
<th>FZ (lbs)</th>
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Conclusions

Continuous track model

- Captures low frequency track vibration in an efficient manner
- Can be married to multibody track models (track/sprocket contact)
- Can be extended to capture track/wheel/terrain interaction