Robust and Worst-Case Design for Vehicle Handling

Nestor Michelena
Hyung Min Kim
University of Michigan

Wei Chen
Raman Garimella
Clemson University
Outline

- Project Goal
- Vehicle and Maneuver Parameters
- Worst-Case Design
  - Rollover metric and worst-case design objective
  - Worst-case design minimax formulation and solution method
  - Results and conclusions
- Robust Design
  - The robust design concept
  - Surrogate ArcSim models
  - Strategies of nonlinear programming-based robust design
  - Robust design metric
  - Results and observations
Project Goal

• To integrate ArcSim models with design optimization techniques
• To implement worst-case and robust design concepts to achieve improved vehicle rollover performance under
  • extreme maneuvers (worst-case design)
  • a wide range of maneuvers (robust design)

• PARTICIPANTS:
  • University of Michigan: worst-case design
  • Clemson University: robust design
Vehicle Specifications

- **M916A1** 3-Axle Tractor (6x6)
- **M870A2** 3-Axle Semitrailer
- 21 rigid body DOF / 91 state variables
- 126,350 lbf GVW
Contributions of Selected Maneuver and Vehicle Parameters to Rollover Metric

- Vehicle parameters with positive effect
- Vehicle parameters with negative effect
- Maneuver parameters with positive effect
- Maneuver parameters with negative effect
Worst-Case Design as an Optimization Problem

• **DESIGN GOAL:**
  - To make a tractor / semitrailer less susceptible to rollover under extreme maneuvers

• **DESIGN VARIABLES (14):**
  - Hitch height (1) and torsional roll stiffness (1)
  - Suspension spring distances (3) and forces (3)
  - Tire spring rates (3)
  - Load distribution (3)

• **MANEUVER PARAMETERS (6):**
  - Steering and braking start/end times (4) and levels (2)
Every vehicle design has an associated extreme maneuver
Worst-Case Design: Objective

**Objective**

**Rollover Metric for Extreme Maneuver**

**Maneuver Parameters:** Braking & Steering

**Min**\textsubscript{vehicle parameters} \quad **Max**\textsubscript{maneuver parameters} \quad Rollover Metric \left( \text{vehicle parameters}, \text{maneuver parameters} \right)

Time (sec) \quad Brake Input Pressure (psi) \quad Time (sec) \quad Wheel Steering Angle (deg)
Worst-Case Design: Method

- Formulate design problem as minimax optimization problem to identify:
  - Extreme maneuver conditions for a given vehicle design
  - Vehicle design parameters that minimize rollover metric for extreme maneuver

- Solve minimax problem using mathematical optimization techniques (semi-infinite sequential quadratic programming)
Minimax Problem Solution Method

Minimax Problem (MMP)

\[
\min_x \left[ \max_y f(x, y) \right]
\]
\[
\text{s.t.} \quad (x, y) \in S_x \times S_y
\]

Semi-Infinite Problem (SIP)

\[
\min x_0
\]
\[
\text{s.t.} \quad x \in S_x \quad \text{and} \quad f(x, y) \leq x_0 \quad \text{for all } y \in S_y
\]

- Discretize \( S_y \): SIP becomes a nonlinear optimization problem
  - with finitely many related constraints,
  - small number of constraints are active at the solution.
- Solve discretized SIP using CFSQP.
- Refine discretization mesh on \( S_y \) and solve SIP again, if needed.
- Solve \( \min_x \) and \( \max_y \) problems in sequence, as needed until convergence.
## Worst-Case Design: Results

<table>
<thead>
<tr>
<th>Vehicle Parameters</th>
<th>Nominal</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Optimum</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Height of hitch above ground (in)</td>
<td>64.0</td>
<td>51.2</td>
<td>76.8</td>
<td>76.8</td>
<td>20.0%</td>
</tr>
<tr>
<td>Hitch roll torsional stiffness (in-lb/deg)</td>
<td>1000000.0</td>
<td>800000.0</td>
<td>1200000.0</td>
<td>800000.0</td>
<td>-20.0%</td>
</tr>
<tr>
<td>Distance between springs on Axle 1 (in)</td>
<td>38.0</td>
<td>30.4</td>
<td>45.6</td>
<td>45.6</td>
<td>20.0%</td>
</tr>
<tr>
<td>Distance between springs on Axle 2/3 (in)</td>
<td>38.0</td>
<td>30.4</td>
<td>45.6</td>
<td>45.6</td>
<td>20.0%</td>
</tr>
<tr>
<td>Distance between springs on Axle 4/5/6 (in)</td>
<td>38.0</td>
<td>30.4</td>
<td>45.6</td>
<td>45.6</td>
<td>20.0%</td>
</tr>
<tr>
<td>Laden load for Axle 1 (lbm)</td>
<td>14425.0</td>
<td>11540.0</td>
<td>17310.0</td>
<td>13704.0</td>
<td>-5.0%</td>
</tr>
<tr>
<td>Laden load for Axle 2/3 (lbm)</td>
<td>25448.0</td>
<td>20358.4</td>
<td>30537.6</td>
<td>24176.0</td>
<td>-5.0%</td>
</tr>
<tr>
<td>Laden load for Axle 4/5/6 (lbm)</td>
<td>20343.0</td>
<td>16274.4</td>
<td>24411.6</td>
<td>21431.3</td>
<td>5.3%</td>
</tr>
<tr>
<td>Axle 1 spring stiffness scale factor</td>
<td>1.0</td>
<td>0.8</td>
<td>1.2</td>
<td>1.18</td>
<td>17.8%</td>
</tr>
<tr>
<td>Axle 2/3 spring stiffness scale factor</td>
<td>1.0</td>
<td>0.8</td>
<td>1.2</td>
<td>0.96</td>
<td>-3.5%</td>
</tr>
<tr>
<td>Axle 4/5/6 spring stiffness scale factor</td>
<td>1.0</td>
<td>0.8</td>
<td>1.2</td>
<td>1.20</td>
<td>20.0%</td>
</tr>
<tr>
<td>Axle 1 tire stiffness (lb/in)</td>
<td>6900.0</td>
<td>5520.0</td>
<td>8280.0</td>
<td>5520.0</td>
<td>-20.0%</td>
</tr>
<tr>
<td>Axle 2/3 tire stiffness (lb/in)</td>
<td>6900.0</td>
<td>5520.0</td>
<td>8280.0</td>
<td>8278.0</td>
<td>20.0%</td>
</tr>
<tr>
<td>Axle 4/5/6 tire stiffness (lb/in)</td>
<td>5174.0</td>
<td>4139.2</td>
<td>6208.8</td>
<td>6209.0</td>
<td>20.0%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Operating Parameters</th>
<th>Nominal</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
<th>Worst Case</th>
<th>Variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steering start time (sec)</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.30</td>
<td>0.0%</td>
</tr>
<tr>
<td>Steering end time (sec)</td>
<td>2.700</td>
<td>2.160</td>
<td>3.240</td>
<td>3.240</td>
<td>20.0%</td>
</tr>
<tr>
<td>Steering level (deg)</td>
<td>100.0</td>
<td>60.0</td>
<td>100.0</td>
<td>100.0</td>
<td>0.0%</td>
</tr>
<tr>
<td>Braking start time (sec)</td>
<td>1.200</td>
<td>1.020</td>
<td>1.380</td>
<td>1.020</td>
<td>-15.0%</td>
</tr>
<tr>
<td>Braking end time (sec)</td>
<td>1.800</td>
<td>1.530</td>
<td>2.070</td>
<td>2.070</td>
<td>15.0%</td>
</tr>
<tr>
<td>Braking level (psi)</td>
<td>120.0</td>
<td>70.0</td>
<td>120.0</td>
<td>120.0</td>
<td>0.0%</td>
</tr>
<tr>
<td>Rollover Metric (deg-sec^{0.5})</td>
<td>66.84</td>
<td></td>
<td></td>
<td>60.39</td>
<td>-9.6%</td>
</tr>
</tbody>
</table>
Worst-Case Design: Results

Vehicle worst-case design extreme maneuver:

- Hitch height
- Hitch roll stiffness
- Distance between springs on Axle 1, Axle 2/3, Axle 4/5/6
- Laden load for Axle 1, Axle 2/3, Axle 4/5/6
- Spring stiffness Axle 1, Axle 2/3, Axle 4/5/6
- Tire stiffness Axle 1, Axle 2/3, Axle 4/5/6
- Steering start time, end time, level
- Braking start time, end time, level
- Worst-case rollover metric
Worst-Case Design: Conclusions

- Worst-case design problems are computationally expensive to solve.

- Models such as
  - reduced order models (ARC area 1) and
  - surrogate models (Wei Chen)

  can facilitate solution by reducing simulation runtimes to evaluate worst-case rollover metric.
The Concept of Robust Design

Given
- Model $f(x, z)$
- Noise factors $(m_z, s_z)$

Find
- Control factors $x$

Satisfy
- System constraints

Objectives
- Bring the mean on target $m_y$
- Minimize the variance $s_y$
Classification of Parameters

Control Factors
- Vehicle Parameters

Operating Parameters

Noise Factors

Responses
- Rollover Metric

ArcSim Simulation
Contributions of Selected Operating and Vehicle Parameters to Rollover Metric

Screening Experiments – 350 Latin hypercube designs

9 Design and 5 noise variables contribute more than 95%.
ArcSim Surrogate Model (I)

From ArcSim

Response Surface Model

\[
R^2 = 0.8775 \quad R^2_{\text{adj}} = 0.8175 \\
R^2_{\text{press}} = 0.5042
\]

Secondary Experiments – 225 Latin hypercube, 121 grid, 17 FFE
The rest of the parameters are set at their norminals.
The use of the response surface models can significantly improve computational efficiency.

Statistical Approach to Robust Design

Given: A range of Maneuver Conditions (ΔZ)

Generation of vehicle and suspension parameters (x)

Simulations of vehicle performance by picking a set of random numbers within the range of maneuver conditions (ΔZ)

Evaluation of Robust Design Metric based on the mean and the variance of handling metric

Maximize Robust Design Metric

Is the convergence achieved?

 Optimization Iterations

Y Stop

N
Bi–Level Robust Optimization Strategy

Given: A range of Maneuver Conditions (ΔZ)

Generation of vehicle and suspension parameters (x)

Worst Case Optimization
Best Case Optimization

Evaluation of Robust Design Metric based on the best and the worst handling metric

Maximize Robust Design Metric

Is the convergence achieved?

Y
Stop

N
Optimization Iterations

The best and worst steering and breaking inputs are identified through sub-optimizations
Robust Design Metric

Maximize \( C_{dk} = \frac{C - \mu_y}{3\sigma_y} \) to 1

Robust Design Metric

(maximize common range)
Critical Rollover Condition

No rollover if simulation stops after 5 secs.

C is taken as 45 deg-sec\(^{1/2}\) based on interpolation.
Results Based On Statistical Approach

Full Noise Range

Reduced Noise Range

Density Function

Rollover Metric

Mean 9.2722  Std Dev 15.2191

Mean 8.1129  Std Dev 4.8033

\( c_{dk} = 0.7825 \)

\( c_{dk} = 2.5599 \)

- end–brake (s): 1.53, 2.07
- steer_level (deg): 60, 100
- brake_level (psi): 70, 100

- end–brake (s): 1.611, 1.989
- steer_level (deg): 66, 94
- brake_level (psi): 77.5, 112.5
Robust Design Results

Vehicle Robust Design

Range of Maneuvers

Dev. from Nominal

-50%
-40%
-30%
-20%
-10%
0%
10%
20%

Hitch height
Hitch roll stiffness
Dist. btw springs on Axle 2/3
Laden load for Axle 4/5/6
Spring stiffness Axle 1
Spring stiffness Axle 2/3
Tire stiffness Axle 1
Tire stiffness Axle 2/3
Steering start time
Braking start time
End time
End time
Level
Level

Robust Design Metric

395%
Robust Design Compared to Baseline

Baseline

Rollover Metric

Mean 14.67030
Std Dev 17.91330

$C_{dk} = 0.5644$

Robust Design (Bi-Level)

Rollover Metric

Mean 8.03539
Std Dev 4.40803

$C_{dk} = 2.7952$
Comparison of Optimal Designs from Different Strategies

Results obtained from the statistical approach and the bi-level approach are very identical.
Observations

- Robust design approach can be used to effectively reduce the impact of “noise” parameters as well as to achieve the optimal vehicle performance.
- Simpler models significantly improve the computational efficiency for robust optimization and provide insight into the problem.
- Both statistical approach and the bi-level optimization strategy are valid for robust design.